Nassau County Interscholastic Mathematics League

Contest #4 Answers must be integers from 0 to 999, inclusive. 2021 – 2022

Calculators are allowed.

Time: 10 minutes **Name: _____**

19. The lengths of the sides of a non-isosceles triangle in increasing order are 5, x, and 15. Compute the sum of all possible integral values of x.

20. Compute the product of the first 12 terms of a geometric sequence whose first term is 1 and whose twelfth term is 2.

19. ☐ 20. ☐

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21. The perimeter of a square is increased by 20 percent. The area of the square is increased by $k\%$. Compute k.

22. If the coordinates of the center of the circle containing points $A(3,-1)$, $B\ (5,-5)$ and $C(9,-3)$ are (p,q), compute $p+q$.

21. ☐ 22. ☐

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23. On a stream with steady current, Lori takes 15 minutes to swim upstream. It takes her only 5 minutes to swim the same distance downstream. When Lori swims, she swims consistently at her fastest pace. How many minutes does it take Lori to <u>float</u> the same distance downstream?

24.. If $f(x) = a \cdot x^6 + b \cdot x^4 + x^3 + x + 13$ and $f(-5) = 11$, compute $f(5)$.

23. ☐ 24. ☐

Solutions for Contest #4

19. The sum of the lengths of any two sides of a triangle is greater than the length of the remaining side. Therefore, $5 + x > 15 \rightarrow x > 10$. Since $5 < x < 15$, x can be 11, 12, 13, or 14. The required sum is **50.**

20. If r is the common ratio, then $a_{12} = a_1 r^{11} = 2 \rightarrow r^{11} = 2 \rightarrow r = 2^{\frac{1}{11}}$. The required product is $2^{\left(0+\frac{1}{11}+\frac{2}{11}+\cdots+\frac{11}{11}\right)} = 2^6 = \mathbf{64.}$

21. If the length of a side of a square is x, then its area is x^2. The side of the enlarged square is $1.2x$ and the area of the enlarged square is $1.44x^2$. So, the area of the original square has been increased by **44** percent.

22. The center of the circle is the intersection of the perpendicular bisectors of two chords. The midpoint and slope of $\overline{AC}$ are respectively, $(6, -2)$ and $-\frac{1}{3}$. The equation of its perpendicular bisector is $y + 2 = 3(x - 6)$. The midpoint and slope of $\overline{BC}$ are respectively, $(7, -4)$ and $\frac{1}{2}$. The equation of its perpendicular bisector is $y + 4 = -2(x - 7)$. Solving the system of equations simultaneously, the coordinates of the center are $(6, -2)$. The required sum is **4**.

23. Lori swims at x miles per minute in still water. The speed of the current is y miles per minute. Lori swims d miles each way. Therefore, $5(x + y) = d$ and $15(x - y) = d$. So, $5x + 5y = 15x - 15y \rightarrow x = 2y$. Thus, $5(2y + y) = d \rightarrow d = 15y$. Since Lori floats at y miles per minute, the required number of minutes is **15.**

24. Start with $f(-5) = a \cdot (-5)^6 + b(-5)^4 - 125 - 5 + 13 = 11$. Since $(-5)^6 = 5^6$ and $(-5)^4 = 5^4$, $a \cdot 5^6 + b \cdot 5^4 = 128$. So, $f(5) = 128 + 125 + 5 + 13 = \mathbf{271}$.